Derivatives, integrals
supplementary exercises

Note: those exercises are not obligatory, they will not be treated during chalkboard practice, and will not be included in any tests.

1. Radioactive decay is a good example of an exponential decay. Number of decays per unit time in a sample is proportional to the amount of nuclei N, which this sample contains. In other words, the rate of decay is proportional to N. Therefore:

$$\frac{dN(t)}{dt} \propto N(t) \tag{1}$$

Notice however that since $N(t)$ is a decreasing function (there is smaller and smaller amount of nuclei due to decay, so its derivative is negative) there should rather be

$$\frac{dN(t)}{dt} \propto -N(t) \tag{2}$$

Finally, if we take λ to be our proportionality constant, we can write an equality instead of proportionality

$$\frac{dN(t)}{dt} = -\lambda N(t) \tag{3}$$

Knowing that the sample contains initially Avogadro's number of atoms, and using equation 3 find the formula for N(t). (Hint: use integration!!). Think about the half-life definition from the secondary school. Knowing N(t), can you derive the formula for it?

2. This is an exercise for those who have a calculator with the possibility to numerically evaluate definite integrals. We can easily try-out the quality of an integration procedure employed in the machine, by calculating the following integral

$$\int_0^1 \sin(ax)dx \tag{4}$$

for the given set of values a:$[1, 10, 20, 40, 100]$. What is the time of calculations taken to evaluate the integrals with increasing a? Does it grow or not? Is the result in agreement with analytically calculated values?